Introduction to the Move On Maths Methods Module

How do you add, subtract, multiply and divide?
What approaches do you use to support learners?
This module will provide an overview of some of the most commonly used strategies for calculation.

This module has been produced to support teachers of numeracy to learners working towards individual learning plan numeracy targets from Entry Level to Level 2, including national accreditation as appropriate to the learner.

The target audience includes, but is not exclusive to, *Skills for Life* teachers, key skills teachers, work-based learning assessors, vocational teachers and classroom assistants, and has been developed in response to feedback from these groups and national consultation with other organisations and projects involved in supporting the development of maths strategies for learners.

It is clear that learners will seek support for number related learning to meet a variety of needs, such as improving their maths skills, promotion in employment, completion of apprenticeship frameworks, support for children at school, and to fill gaps in their CVs.

Methods for completing the four basic calculations: addition, subtraction, multiplication and division, are varied and have been developed over time to include some different approaches. As trainers, it can sometimes cause confusion if learners use an approach that is unfamiliar or not previously encountered, and this module has been developed to support and explain some of these strategies.

Experts in the field have been involved in the development of this module, including those from primary school phase and *Skills for Life* teachers.

This module will provide an overview of some of the most commonly used strategies for calculation, but there are many variations on a theme, and deliverers may find that they will need to adapt their understanding of the main approaches to meet the needs of their learners.

As with any number development support that a learner may need, numeracy subject specialist teachers are equipped to provide advice and guidance on number learning, and this module does not seek to replace the need or desirability of consulting with the subject specialists on any number development issues that may arise when working with learners.

The module includes a session plan and delivery methodologies, together with resource sheets and presentations. Suggestions for learner activities are also indicated and these are supported by the participant activities to which they are attached.

Suggested timings for the activities have been indicated, and a 15 minute allowance each half day session for a tea/coffee break included, but the actual point at which these are taken is at the discretion of the trainer.

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Course outline

Course Move On Maths Module	**Module** Methods of calculation	**Duration** 6 hours	**Week**	**Venue**
Topic Overview of approaches to calculations of addition, subtraction, multiplication and division.		**Starting point** The group is supporting learners in number from Entry Level to Level 2 in the National Qualifications Framework.		

Aims:

- To introduce participants to some of the more common approaches learners may use to complete calculations in addition, subtraction, multiplication and division.
- To enable participants to consider the importance of the language of number to support learners in their approaches to these calculations.
- To introduce the concept of identifying and analysing errors in these calculations.

Objectives

Participants will be able to:

1 consider their own and others' experiences of approaches to completing calculations in addition, subtraction, multiplication and division
2 identify some of the more common strategies and approaches to completing these calculations
3 develop an understanding of the importance of the language of number in approaches to supporting learners in these calculations
4 identify some of the errors learners may make when completing these calculations.

Session plan

Session: Methods of calculation

Timing	Content	Method	Resources	Evaluation
5 mins	**Welcome, introduction and housekeeping**	Trainer to welcome participants and introduce the module. Housekeeping arrangements detailed, e.g. coffee breaks, lunch, fire drills and exits.	OHP slide 2	
15 mins	**Activity 1 – Icebreaker: The language of maths**	Small group activity. Participants are requested to match the sets of cards. One set contains some maths terminology (taken from the Adult Numeracy Core Curriculum glossary), the other set contains the descriptions of the maths terms. Feedback and discussion, highlighting the need to understand and explain the language of maths to learners. This is to ensure that common terminology is used during sessions.	Sets of cards with maths terms and descriptors Activity sheet 1 – Adult Numeracy Core Curriculum	
10 mins	**Activity 2 – The language of maths continued**	Small group activity (five groups). Allocate one of the topics (the four rules and the 'do a question') to each group. Each group to list as many different ways of saying the topic they have been allocated as possible. Small group offers feedback of their ideas to the whole group. Handout 1 to summarise. Emphasise the use of language and the confusion it can cause learners using slide 4 as a prompt.	OHP slides 3 and 4 Handout 1	

Timing	Content	Method	Resources	Evaluation
15 mins	**Activity 3 – Mental maths: 'Follow Me' game**	This is an activity to enable participants to 'warm up' their mental maths skills. This can also be used with learners (taken from the Adult Numeracy Core Curriculum training). Ask one of the participants to time the activity (regular exercises like this should improve speed as well as number skills). All the cards will need to be handed out, so if there are more participants than cards, the cards will have to be shared. Highlight that some of the maths language from the previous activity may be used. Trainer starts the activity to demonstrate and will therefore end it, as it is a complete circle activity. After the activity, discussion to identify skills used to complete it.	'Follow Me' cards (cut up from Handout 2)	
40 mins	**Activity 4 – Addition**	Trainer to introduce acronym for these sessions: **TERPA** **T**ry **E**xplore **R**einforce **P**ractise **A**ssess		

Timing	Content	Method	Resources	Evaluation
		The next four sessions will follow a similar pattern: 1 (**T**ry) participants to complete given calculation using own method, then volunteer to demonstrate method. 2 (**E**xplore) trainer to model method not demonstrated (probably modern method – number lines, etc). 3 (**R**einforce) trainer to check understanding using handout. 4 (**P**ractise) participants to practise with partner – talking through the method. Reverse roles. 5 (**A**ssess) participants to try practice questions from Move On practice tests CD-ROM. **Try** - Trainer to write an addition task on flip-chart paper, e.g. 358 + 837. Ask participants to complete the calculation by a method of their own choosing. Ask for a volunteer to demonstrate his/her method, then ask if others did it differently or know a different way, and encourage further demonstrations. **Explore** - Trainer to demonstrate and explain another 1 – 2 methods which have not been shown (taken from the handout), using the flip-chart again. **Reinforce** - Distribute Handout 3 detailing all the methods and ask them to add on any other methods which have been demonstrated (if any). Look through the handout together and invite questions/comments on the various methods.	Handout 3 Handout 4 Flip-chart or whiteboard and markers Paper-based practice test from the *Move On with a National Qualification Toolkit* and computers and CD-ROMs of the same test. Trainer notes 1	

Timing	Content	Method	Resources	Evaluation
		Practise - Ask participants to pair up. Using Handout 4, each pair should demonstrate the same calculation using the three different methods. Trainer to suggest that they attempt methods which are completely new to them where possible. Encourage discussion of the new methods within the pair, including what they find harder/easier/confusing/etc. about the new methods attempted. **Assess** - Ask participants to apply their learning to practice questions taken from the Move On practice tests CD-ROM. See Trainer notes 1 on appropriate questions to use. Note: This will require participants to practise the skill of converting a problem in words into an addition task.		
50 mins	**Activity 5 - subtraction**	Review the TERPA process which will be used for this next session. **Try** - Trainer to write a subtraction task on flip-chart paper, e.g. 832 - 475. Ask participants to complete the calculation by a method of their own choosing. Ask for a volunteer to demonstrate his/her method, then ask if others did it differently or know a different way and encourage further demonstrations. **Explore** - Trainer to demonstrate and explain another one or two methods which have not been shown (taken from the handout), using the flip-chart again.	Handout 5 Handout 6 Flip-chart or whiteboard and markers Paper-based practice tests from the *Move On with a National Qualification Toolkit*. Computers and CD-ROMs of the same test Trainer Notes 2	

Timing	Content	Method	Resources	Evaluation
		Reinforce - Distribute Handout 5 detailing all the methods and ask them to add on other methods which have been demonstrated (if any). Look through the handout together and invite questions/comments on the various methods. **Practise** - Ask participants to pair up. Using Handout 6, each participant should demonstrate his/her preferred method to the other person and then attempt the same calculation using two different methods to the original method. Trainer to suggest that they attempt methods which are completely new to them where possible. Encourage discussion of the new methods within the pair, including what they find harder/easier/confusing/etc. about the new methods attempted. **Assess** - Ask participants to apply their learning to practice questions taken from the Move On practice tests CD-ROM. See Trainer notes 2 on appropriate questions to use. Note: This will require participants to practise the skill of converting a problem in words into a subtraction task.		
55 mins	**Activity 6 - Multiplication**	Review the TERPA process which will be used for this next session. **Try** - Trainer to write a long multiplication task on flip-chart paper, e.g. 67 x 42. Ask participants to complete the calculation by a method of their own choosing. Ask for a volunteer to demonstrate his/her method, then ask if others did it differently or know a different way and encourage further demonstrations.		

Timing	Content	Method	Resources	Evaluation
		Explore - Trainer to demonstrate and explain one or more methods which have not been shown (taken from the handout), using the flip-chart again. It may be particularly useful at this stage to demonstrate the gypsy method, if appropriate. **Reinforce** - Distribute Handout 7 detailing all the methods and ask them to add on any other methods which have been demonstrated (if any). Look through the handout together and invite questions/comments on the various methods. **Practise** - Ask participants to pair up. Using Handout 8, each participant should demonstrate his/her preferred method to the other person and then attempt the same calculation using at least two methods which are different from the original method. Trainer to suggest that they attempt methods which are completely new to them where possible. Encourage discussion of the new methods within the pair, including what they find harder/ easier/confusing/etc. about the new methods attempted. **Assess** - Ask participants to apply their learning to practice questions taken from the Move On practice tests CD-ROM. See Trainer notes 3 on appropriate questions to use. Note: This will require that participants practise the skill of converting a problem in words into a multiplication task.	Handout 7 Handout 8 Flip-chart or whiteboard and markers Paper-based practice tests from the *Move On with a National Qualification Toolkit*. Computers and CD-ROMs of the same test. Trainer notes 3	

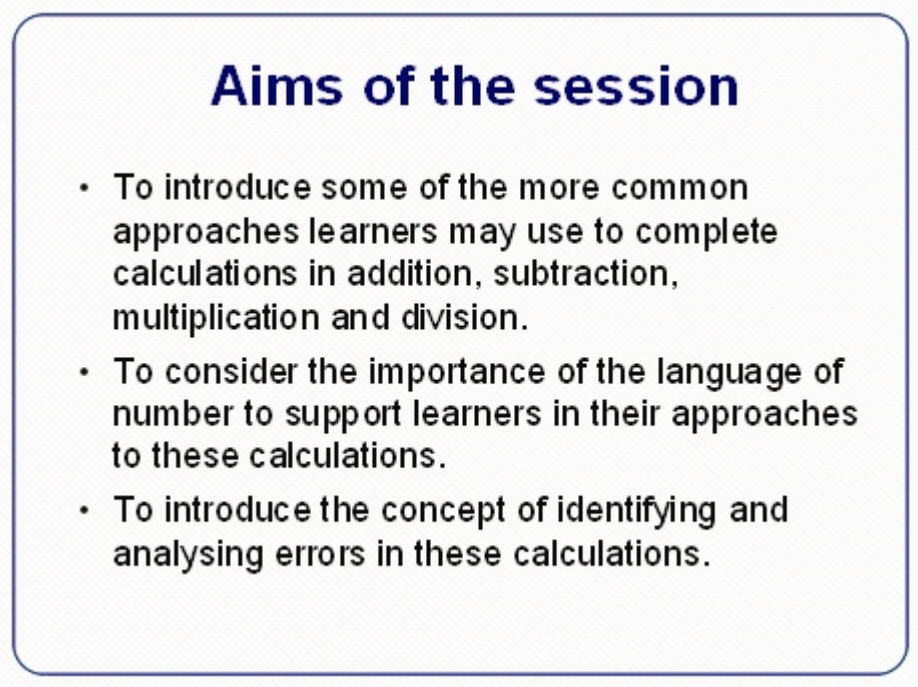
Timing	Content	Method	Resources	Evaluation
60 mins	**Activity 7 - Division**	Review the TERPA process, which will be used for this next session. **Try** - Trainer to write a long division task on flip-chart paper, e.g. 2282 ÷ 14. Ask participants to complete the calculation by a method of their own choosing. Ask for a volunteer to demonstrate his/her method, then ask if others did it differently or know a different way and encourage further demonstrations. **Explore** - Trainer to demonstrate and explain another one or two methods which have not been shown (taken from the handout), using the flip-chart again. **Reinforce** - Distribute Handout 9 detailing all the methods and ask them to add on any other methods which have been demonstrated (if any). Look through the handout together and invite questions/comments on the various methods. **Practise** - Ask participants to pair up. Using Handout 10 each pair should demonstrate the same calculation using at least two new methods. Trainer to suggest that they attempt methods which are completely new to them where possible. Encourage discussion of the new methods within the pair, including what they find harder/easier/ confusing/etc. about the new methods attempted. **Assess** - Ask participants to apply their learning to practice questions taken from the Move On practice tests CD-ROM. See Trainer notes 4 on appropriate questions to use. Note: This will require participants to practise the skill of converting a problem in words into a division task.	Handout 9 Handout 10 Flip-chart or whiteboard and markers Paper-based practice tests from the *Move On with a National Qualifications Toolkit*. Computers and CD-ROMs of the same test. Trainer notes 4	

Timing	Content	Method	Resources	Evaluation
30 mins	**Activity 8 - FAQs (frequently asked questions)**	Small group activity to identify appropriate responses to FAQs on Handout 11 and any new suggestions of FAQs from own learners. Whole group activity, facilitated by trainer, to agree responses to FAQs. Trainer to get list typed up and sent to Move On to be included on web site.	Paper and coloured pens Example FAQs and answers – Handouts 11 and 12 (answers)	
30 mins	**Activity 9 - Error analysis**	Trainer to introduce the activity with Slide 5 and to highlight the key types of errors learners make. Using Slide 6, talk through the examples of errors and refer participants to Handout 13 In small groups, participants to complete analysis of one of the errors in the calculations on Handout 14. Feedback briefly and give out Handout 15 for participants to take away. Mention that error analysis information and examples have been taken from the *Skills for Life* diagnostic assessment resources/primary numeracy strategy/key skills support programme. There are further examples of learner errors in the *Skills for Life* numeracy diagnostic assessment.	OHP slides 5 and 6 Handout 13 Handout 14 Handout 15	

Timing	Content	Method	Resources	Evaluation
10 mins	**Activity 10 - Summing up and action planning** **'Try it out with your learners' activities**	Trainer to sum up session, referring back to aims and highlighting learning points. Trainer to show Slide 2 Participants to complete action plan on Handout 16 to identify further development needs or activities in their own organisations Refer participants to Handout 17 which contains some ideas for activities participants could do with their learners.	OHP slide 7 Handout 16 Handout 17	

Welcome, introduction and housekeeping

Method:



- Welcome participants and introduce yourself (and your co-trainer, if appropriate).

- Explain the format of the day, the arrangements for fire drill and whether a fire alarm is scheduled, times of breaks and lunch, the location of toilet facilities, etc.

- Introduce the aims of the session, using Slide 2 as support. Explain the background to the development of the module as detailed in the introduction, i.e.:

  – A resource developed by the Move On project.
  – Developed in response to feedback from the field.
  – Developed by experts in the field from primary, secondary and FE.
  – Not a replacement for advice, guidance and expertise of numeracy specialists.
  – Support materials for a range of staff supporting learners in number skills.

Activity 1 – Icebreaker: The language of maths

Method:

- Copy the cards on Activity Sheet 1 onto coloured card and cut up into sets. You will need to prepare sufficient sets for each group of participants to have one.
- Introduce the activity as an opportunity to look at some of the language of maths/number, starting with some of the terminology from the Adult Numeracy Core Curriculum.
- Ask participants to work in small groups, and hand out a set of cards to each group. (Make sure each set of cards is well shuffled!)
- Explain that there is a definition card for each maths term, and that each group should match up their set of cards.
- Ask the whole group to offer feedback on their thoughts on the activity, highlighting the need to understand and explain the language of maths to learners to ensure that common terminology is used during sessions.
- Highlight that this activity could be used with learners as suggested in the 'Try it out with your learners!' activity below. Refer participants to Handout 17, which contains all the 'Try it out with your learners!' activities.

Try it out with your learners! Activity 1

Use the meaning cards with your learners in a matching activity. Focus on number words and their associated meanings that learners will need in their core course work.

Use the activity to introduce learners to a new topic at the start of a session. You could also use this type of activity to introduce learners to new words and definitions in any topic or area of study.

Activity sheet 1 – The language of maths

One of the symbols of a number system.	**digit**
To arrive at a rough answer by calculating with suitable approximations for numbers.	**estimate**
Any of the positive or negative whole numbers, including zero.	**integer**

A result that is not exact but sufficiently close to be useful in a practical context.	**approximation**
An operation * is commutative if a*b = b*a. Addition and multiplication are commutative, subtraction and division aren't.	**commutative**
Relating to base 10.	**decimal**

The value of a digit that relates to its position or place in a number.	**place value**
Has exactly two factors, itself and 1.	**prime number**
A comparison of quantities of the same kind.	**ratio**

A pair of numbers with a particular total.	**number bond**
A means of combining numbers, sets or other elements, e.g. addition and subtraction.	**operation**
A symbol used to denote an operation.	**sign**

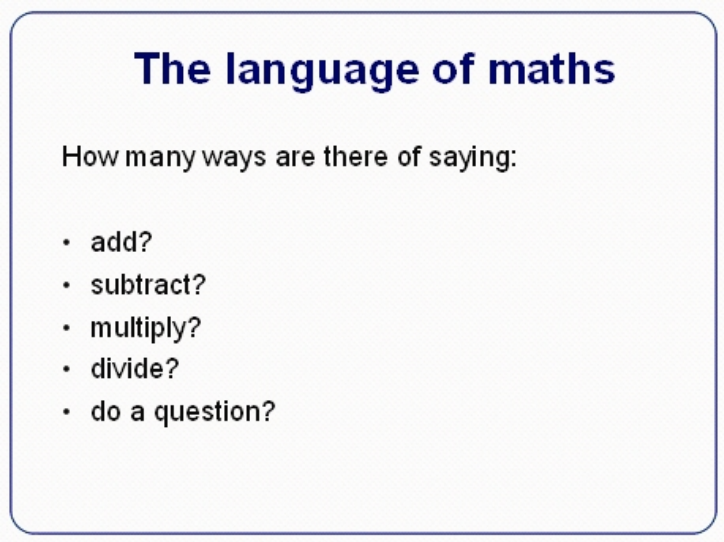
Activity 2 – The language of maths continued

Method:

- Introduce the activity as a follow on from the previous activity - looking at the language of maths. We all have our own ways of saying maths operations and asking learners to solve a problem. This can cause great confusion for learners and teachers alike! It is essential that we talk to learners in the same language and this activity will enable everyone to think about the language of maths they use and what their learners may be familiar with.

- Divide the whole group into five smaller groups. (If you have a large number of participants, you may need to allocate one or two of the topics twice.) Allocate one of the following five areas to each group:
  – add
  – subtract
  – multiply
  – divide
  – do a question.

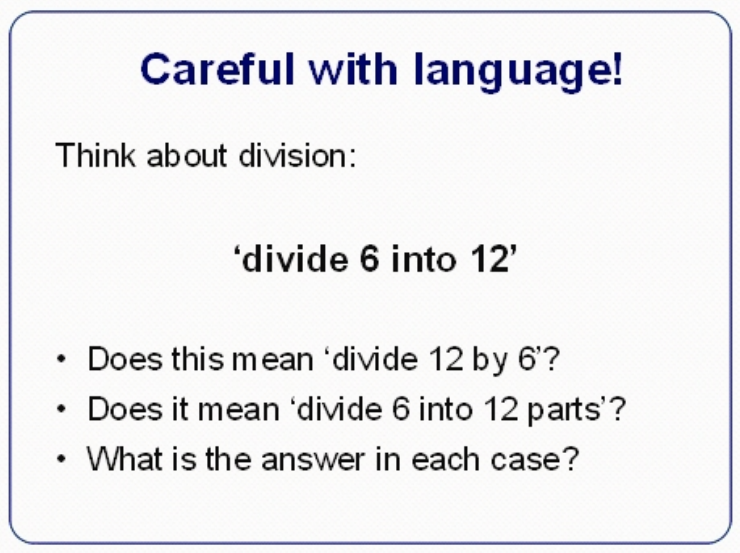
- Use Slide 3 to introduce the task. Each group is to discuss and note down how many different ways there are of saying their allocated topic e.g. for add – sum, total, plus, etc.



- Ask each group in turn to list all the ideas they came up with, and suggest that anyone can contribute additional ideas at the end of each group's feedback. Refer participants to Handout 1, which has a list, not exhaustive by any means, of the different ways of saying the five topics.

- Summarise this and the previous activity highlighting that the language of maths can be very confusing for learners, especially those with learning difficulties and disabilities.

- Use Slide 4 to support this and encourage any further discussion – this slide highlights some of the pitfalls and problems when supporting number learning with language and communication.



- Mention that this will also link into the next activity – mental maths – so participants need to be on their toes ready to use the language they have been introduced to already. Refer participants to the 'Try it out with your learners!' activity on Handout 17.

Try it out with your learners! Activity 2

The questions in the adult numeracy tests do not ask learners to complete straightforward 'sums', but require them to be able to complete number problems couched in a range of terms such as those identified in the previous activities.

Use a similar activity on the language of maths with your learners to generate a discussion on the language they use in their number calculation methods.

This will provide them with an opportunity to think about the different ways problems can be presented to them in their work and in the adult numeracy tests.

Handout 1 – The language of maths

Add: addition, more, more than, make, and, plus, greater, count on, increase, sum, total, altogether, score, double, how many more to make . . .?

Subtract: take, take away, minus, subtraction, decrease, leave, how many are left? how many are left over?, count back, less, difference between, how many more is . . . than . . .?, how many fewer is . . . than . . . ?, how many less is . . . than . . . ?, how much more is . . .?, how much less is . . .?

Multiply: of, lots of, groups of, times, by, product, multiplication, multiplied by, multiple of, once, twice, three times (etc.), ten times as big . . . (etc.), repeated addition, array, double, nothing at all (as in 'four threes').

Divide: halve, share, share equally, equal groups of . . ., divided by, division, divisible by, remainder, factor, inverse, quotient, shared by, shared between, goes into, how many times . . .

Do a question: this is almost endless! Some examples: solve, simplify, evaluate, find the . . ., calculate, draw, estimate, approximate, work out, find x, rearrange, write down, complete, convert, what is . . ., mark, sketch, change, how much . . .?, how many . . .?, predict, describe, investigate, choose, decide, collect, use, make, construct, define, identify, explain, give an example of . . ., show how you . . ., justify, record, present, interpret, finish, join (up), ring, tally, prove.

Useful web site: Maths language flashcards can be downloaded from:

http://www.standards.dfes.gov.uk/numeracy/publications/resources/vocab_flashcards/366685 (Key Skills 1 and 2)

http://www.standards.dfes.gov.uk/keystage3/respub/ma_vocab_flash
(Years 7, 8 and 9)

Activity 3 – Mental maths: 'Follow Me' game[1]

Method:

- Copy the 'Follow Me' cards on Handout 2 onto thin card and cut up. You will need to have sufficient cards for each participant in the group. If there are more than 14 participants then learners can share cards (but it is important that all cards are used).

- Explain that this is a mental maths starter and enables participants to use some of the key mathematical vocabulary highlighted in the previous activities.

- Select one set of the cards to use and distribute to participants. Each card contains one answer and one question. It may be helpful for you to start the ball rolling by asking the first question. The person with the answer on their card calls it out, and then asks the question that follows from their card, and so the activity continues. Whoever starts will also be the person who finishes, as the game process forms a complete circle. Once the starter answers, the game is complete and they call 'Stop' to end the game.

- Mention that this activity is taken from the Adult Numeracy Core Curriculum Training, which contains an additional set of number cards.

Try it out with your learners! Activity 3

Use the 'Follow Me' cards with learners, focussing on a particular area or topic relevant to the teaching and learning planned.

Time the sessions to encourage learners to speed up their mental maths, as this will help them in their course work and in the adult numeracy tests.

To encourage learner involvement, you could set them a task to develop a set of 'Follow Me' cards to use in whole group sessions.

[1] This activity is taken from the *Adult Numeracy Core Curriculum Training*

Handout 2 – Follow me cards

It's four p.m. Dinner is served in two hours. What time will this be?	It's quarter to eight. I have to be at work by 20 to nine. How long have I got?
It's six o'clock. Half an hour ago I was at work. What time was that?	You've got 55 minutes. At three o'clock I go to the shop to get a chocolate bar. I am back at my desk by twenty to four. How long did it take me?
It's half past five. My bus leaves in three and a half hours. What time does it leave?	40 minutes. How many hours and minutes is double this time?
It's nine o'clock. *World Snooker* starts in ten minutes. What time will I need to turn on the TV?	One hour 20 minutes. I go to the pub at nine at night and chucking out time is at ten past eleven. How much drinking time have I got?
It's ten past nine. I will go to bed in 50 minutes. What time will that be?	Two hours and ten minutes. This is how long my train journey should take me, but the train is delayed by 70 minutes. How long will the journey have taken altogether?
It's ten o'clock at night. If I have nine hours' sleep what time will I wake up?	Three hours and 20 minutes. I've got 45 minutes until the *Evening News* starts at half past six. What time is it now?
It's seven o'clock in the morning. I need three-quarters of an hour to get ready for work. What time will I be ready?	It's quarter to six. 1¾ hours ago I was at work. What time was this?
I have a pound. I spend 25 pence. How much have I got left?	I have 85 pence. I find ten pence. How much have I got now?
It costs £2.80. How many ten pence pieces would I need to make up this amount?	I need 50 pence. If this is the change I have been given from £3.25, how much did I spend?

You would need twenty-eight 10 pences. How much are twenty-nine 5 pences worth?	I have 95 pence. I want to pay for an hour's parking which costs £1.50. How much more money will I need?
I have 75 pence. This is the change I get from £5.00 when I buy a box of chocolates. How much was the box?	I spent £2.75. If I buy three cans of cola for 45 pence each, how much money will I have left?
It was £4.25. I spend 50 pence. How much have I got left?	I have 55 pence. If I buy two newspapers costing 26 pence each, how much money will I have left?
I have £1.45. If I buy two tins of beans at 30 pence each, how much money will I have left?	I have £1.40. The magazine I want to buy costs double this. How much will this cost?
I have £3.75. I want to buy a Big Mac meal which costs £4.25. How much more money do I need?	I have three pence. I find 97 pence in my other jacket pocket. How much have I got?

Activity 4 – Addition

Method:

- Introduce the acronym for the next four activities, which will look at some of the common approaches to addition, subtraction, multiplication and division, as follows:

  TERPA
  Try
  Explore
  Reinforce
  Practise
  Assess

- Explain that the next four activities will follow a similar pattern:

  (**T**ry) participants to complete the given calculation using their own method, then volunteer to demonstrate the method.
  (**E**xplore) trainer to model the method not demonstrated (probably modern method such as number lines).
  (**R**einforce) trainer to check participants' understanding using the handout.
  (**P**ractise) participants to practise with their partner – talking through the method. Reverse roles.
  (**A**ssess) participants to try the practice questions from Move On practice tests CD-ROM.

- **Try**
  - Write an addition task on flip-chart paper, e.g. 358 + 479 and ask participants to complete the calculation by a method of their own choosing.
  - Ask for a volunteer to demonstrate his/her method, then ask if others did it differently or know a different way, and encourage further demonstrations. You may need to check that what is being said by volunteers and comment as necessary!

- **Explore**
  - Pick one to two other methods which have not been shown and demonstrate and explain them (the handout is available as support) on the flip-chart or whiteboard. Check that participants are comfortable with and understand the different methods demonstrated.

- **Reinforce**
  - Distribute Handout 3 detailing all the methods and ask participants to add on any other methods demonstrated (if any).
  - Look through the handout together and invite questions/comments on the various methods. You could elicit from participants what approaches they have come across in their work with learners and, if time permits ask them to demonstrate.

- **Practise**
  - Ask participants to pair up. Using Handout 4, each pair should demonstrate the same calculation using the three different methods. Suggest that they attempt methods which are completely new to them where possible. Encourage discussion of the new methods within the pair, including what they find harder/easier/confusing/etc about the new methods attempted.
  - Note: Remind participants to use the correct terminology.

- **Assess**
  - Ask participants to apply their learning to practice questions taken from the Move On practice tests CD-ROM.
  - Note: This will require participants to practise the skill of converting a problem in words into an addition task and link back to the language of maths work in earlier activities.

- **Try it out with your learners!**
  - See ‘Try it out with your learners! Activity 4’ for an addition game.

Handout 3 – Addition

Standard method involving carrying from units to tens, tens to hundreds etc.

	H	T	U
	3	5	8
+	4	7	9
	8	3	7
	1	1	

Commentary:

8 and 9 is 17

7 is written or 'goes' in the units column and the TEN (NOT a 1) is placed under the line in the tens column.

10 and 70 is 80
80 and 50 is 130

The 30 is written in the tens column and the hundred goes under the line in the hundreds column.

100 and 400 is 500
500 and 300 is 800

It is important that learners are confident about the value of the digits they are working with, especially in the 'carrying' procedure.

Partitioning

Using a hundred, tens and units number

	358	is	300	+	50	+	8					
+	479	+	400	+	70	+	9					
	837		700	+	120	+	17	=	820	+	17	= 837

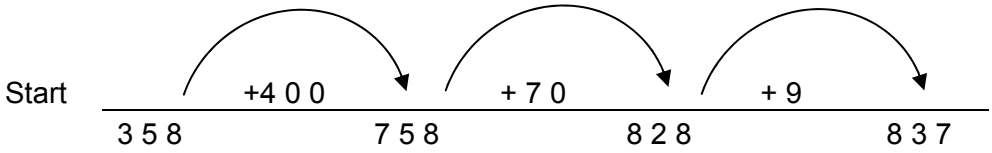
We are aiming for learners to use an efficient standard method, but they will be at different stages on their route to this efficient method. What is important is that they understand what they are doing at each stage. As they gain confidence they can move onto the next stage.

Although the example uses a hundred, tens and units number, this method can be applied with larger numbers.

Using a number line

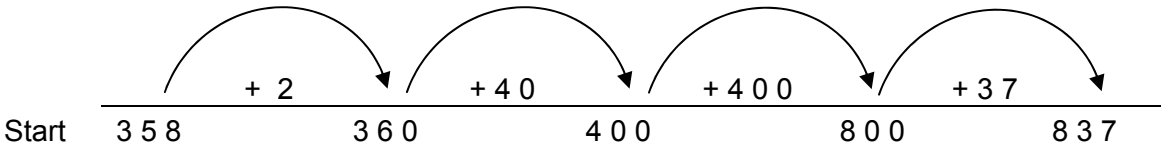
Your learners may have had experience of using number lines from the age of four or five. Obviously the numbers on the lines will have been much smaller. They will have used vertical and horizontal number lines, but this example looks at the problem of using a horizontal number line.

Using a horizontal number line this is how learners would record it.



- The learners will count on in jumps of hundreds, tens and units.
- From 358 if they add 400 they arrive on 758.
- By adding 70 they come to 828. Some learners may have to break this down into seven steps of +10. This could indicate that they would benefit from more practice bridging through a hundred.
- From 828 they can add 9 to reach 837.

This question could have been tackled by working from the least significant digit, the eight units, first.



- 358 plus gives us 360.
- A further jump of 40 lands us on 400.
- By moving on 400 more we arrive at 800.
- Finally by adding 37 we reach 837.

Learners may be in-between two methods; e.g. they could be confident when partitioning but need support when attempting to record using columns.

Addition problems in words

It is important that learners are taught how to convert a problem presented in words into an addition task which can be solved by one of the methods above.

For example, in one class twelve children stayed for dinner on Monday, six more stayed on Tuesday. On Wednesday 16 children stayed for dinner and on Thursday there were five fewer. On Friday there were the same number as on Monday. What is the total of number of children who stayed to dinner during the week?

$$\begin{array}{rr} & 12 \\ & 6 \\ & 16 \\ & 11 \\ & 12 \\ + & 57 \\ & \overline{}^{2} \end{array}$$

It is always advisable to encourage learners to estimate the answer in order to check that the answer obtained is reasonable. The use of inverse operations could also be used to check the answer. This helps in establishing the link between addition and subtraction.

The methods demonstrated above include the ones most commonly encountered and those suggested in the Adult Numeracy Core Curriculum, but it is possible that you will encounter additional methods or variations on these methods.

Handout 4 – Addition examples

With a partner, work out the following question using the three different methods.
Talk it through as your partner listens, and then reverse roles to tackle it another way.

Standard method

	H	T	U
	5	6	4
+	3	9	8

Partitioning

Using a number line

Trainer notes 1 – Practice questions

Appropriate questions for addition from the practice tests are referenced below. There are many questions using the four rules of calculation on the Move On practice tests CD-ROM, but three from each of Level 1 and Level 2 have been indicated which focus on using addition.

It would be helpful for participants to obtain a copy of the CD-ROM (available to order on the Move On web site or from DfES Publications on 0845 60 222 60 quoting reference NTT/PT04/CD) to enable them to practise the various approaches to calculations and to use with their learners.

Level 1

Practice test	**Question**
Test A	Q 16
Test C	Q 3
Test D	Q 24

Level 2

Note: there are many questions in the Level 2 tests which require two stage calculations. The questions indicated below focus mainly on addition, although may require an understanding of other number related skills.

Practice test	**Question**
Test A	Q 3
Test C	Q 13
Test D	Q 5

Activity 5 – Subtraction

Method
Quickly review the TERPA acronym and its process, explaining that the same format will be used to look at subtraction.

- **Try**
  - Write a subtraction task on flip-chart paper, e.g. 832 - 475 and ask participants to complete the calculation by a method of their own choosing.
  - Ask for a volunteer to demonstrate his/her method, then ask if others did it differently or know a different way, and encourage further demonstrations. You may need to check that what is being said by volunteers is correct and comment as necessary!

- **Explore**
  - Pick one to two other methods that have not been shown and demonstrate and explain them (the handout is available as support) on the flip-chart or whiteboard. Check that participants are comfortable with and understand the different methods demonstrated.

- **Reinforce**
  - Distribute Handout 5 detailing all the methods and ask participants to add on any other methods demonstrated (if any).
  - Look through the handout together and invite questions/comments on the various methods. You could elicit from participants what approaches they have come across in their work with learners and, if time permits, ask them to demonstrate.

- **Practise**
  - Ask participants to pair up. Using Handout 6, each pair should demonstrate the same calculation using three different methods. Suggest that they attempt methods that are completely new to them where possible. Encourage discussion of the new methods within the pair, including what they find harder/easier/confusing etc. about the new methods attempted.
  - **Note:** Remind participants to use the correct terminology.

- **Assess**
  - Ask participants to apply their learning to practice questions taken from the Move On practice tests CD-ROM.
  - **Note:** This will require participants to practise the skill of converting a problem in words into a subtraction task, to link back to the language of maths work in earlier activities.

- **Try it out with your learners**
  - See ‘Try it out with your learners! Activity 5’ for a subtraction game.

Handout 5 – Subtraction

'Borrow one and pay back'

Depending on the age of the learners, the method used for subtraction may be 'borrow and pay back'

If, when explaining the methods used, a participant demonstrates solving the problem using borrowing and paying back, explain that this method is no longer taught in schools. It may be necessary to discuss the method, checking that the correct vocabulary is used, and explaining the importance of this.

In this example the language is worth listening to because it may not actually make sense, especially to a learner who is struggling to understand.

$$\begin{array}{r} 8\ 3\ {}^{1}2 \\ -\ 4\ 7_{1}\ 5 \\ \hline \quad\quad 7 \end{array}$$

$$\begin{array}{r} 8\ {}^{1}3\ {}^{1}2 \\ -\ 4_{1}\ 7_{1}\ 5 \\ \hline 3\ 5\ 7 \end{array}$$

Commentary:
5 from 2 you can't do
Borrow 1
5 from 12 is 7
Pay it back (at the bottom of the next column)
7 and 1 is 8
8 from 3 you can't do
Borrow 1
8 from 13 is 5
Pay it back (at the bottom of the next column)
4 and 1 is 5
5 from 8 is 3

Decomposition

$$\begin{array}{r} 8\ {}^{2}\cancel{3}\ {}^{1}2 \\ -\ 4\ 7\ 5 \\ \hline \quad\quad 7 \end{array}$$

$$\begin{array}{r} {}^{7}\cancel{8}\ {}^{12}\cancel{3}\ {}^{1}2 \\ -\ 4\ 7\ 5 \\ \hline 3\ 5\ 7 \end{array}$$

Commentary:
2 take away 5 we can't do
If we change the 30 into 20 plus 10 units we can put the 10 units in the units column
Then we have 12 (ten and 2) take away 5 is 7
Write 7 in the units place
Looking at the tens column we have 20 take away 70 which we can't do
We can change the 800 into 700 and 10 lots of 10
120 take away 70 is 50 and then we write 50 in the tens column
700 take away 400 is 300

This example involved decomposition of both hundreds and tens. If this is causing problems it may be necessary to go back to work on questions that only involve decomposition of either hundreds or tens until the learner feels more confident.

Partitioning

Breaking down or splitting the given numbers to make them easier to handle is a popular method. This is called partitioning.

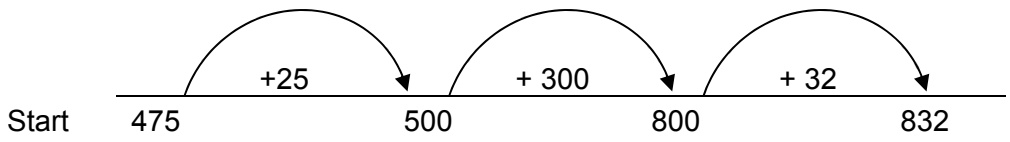
	832	is	800	+	30	+	2
-	475		- 400	+	70	+	5
	357						
		is	800	+	20	+	12
			- 400	+	70	+	5
		is	700	+	120	+	12
			- 400	+	70	+	5
			300	+	50	+	7

- 832 is partitioned into 800 plus 30 plus 2.
- 475 is also partitioned into 400 plus 70 plus 5.
- It is important to keep the hundreds, tens and units columns in line.
- Starting with the units it is impossible to take 5 away from 2 so the tens are repartitioned to read 20 plus 10 and the 10 is added to the 2 giving 12.
- It is now possible to subtract the units but a problem occurs in the tens column.
- To solve this, the hundreds are repartitioned as 700 plus 100 and the 100 is added to the 20 giving 120.
- The question can now be completed.

As stated earlier when looking at addition, we are aiming for learners to be confident using an efficient standard method. If learners are experiencing difficulty it may be useful to go back to a process that they were confident with. They may have tried to move on too quickly before thoroughly acquiring a concept properly or attempted too big a step.

Using a number line

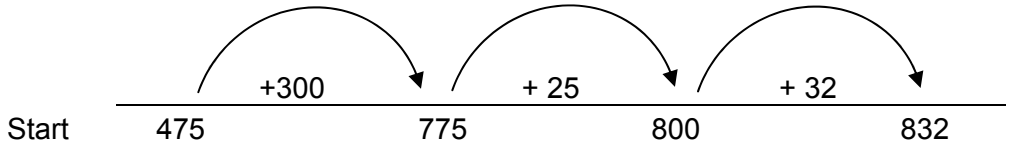
Another way to solve this problem involves the use of a number line. Using a number line, this is how learners might record it:



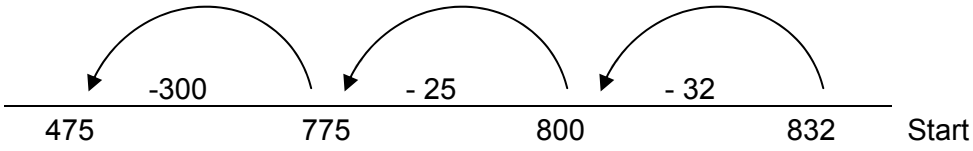
If the learners move from 475 to 832 they are in effect counting up from the smaller to the larger number. This highlights the importance of them understanding the relationship between addition and subtraction.

It does not matter whether the least or most significant digit is used first but sometimes one way is easier than another.

In this example the learner would need to be aware that by adding 400 to 475 may give the correct number of hundreds but 875 is bigger than the original number.



Some learners may prefer to start at the other end of the number line with 832 and count backwards until they arrive at 475.



If the learner was counting on to find the difference, they could be encouraged to record it like this:

832		
475		
5	to	480
20	to	500
300	to	800
32	to	832
357		

If they were counting back the recording would look like this:

832		
475		
32	to	800
25	to	775
300	to	475
357		

This may help to reinforce the importance of lining up the units in the units column and the tens in the tens column etc. The same form of recording could be used with addition questions using the number line.

Subtraction problems in words

It is important that learners are taught how to convert a problem presented in words into a subtraction task, which can be solved by one of the methods above.

For example, a flag seller had 100 flags to sell. At the end of the day he had eleven left. How many flags had he sold?

100		
11		
9	to	20
80	to	100
89		

It is always advisable to encourage learners to estimate the answer in order to check that the answer obtained is reasonable. The use of inverse operations could also be used to check the answer. This helps in establishing the link between addition and subtraction.

The methods demonstrated above include the ones most commonly encountered and those suggested in the Adult Numeracy Core Curriculum, but it is possible that you will encounter additional methods or variations on these methods.

Handout 6 – Subtraction examples

With a partner work out the following question using three different methods. Talk it through as your partner listens and then reverse roles to tackle it another way.

653 - 274

Method 1

Method 2

Method 3

Trainer notes 2 – Practice questions

Appropriate questions for subtraction from the practice tests are referenced below. There are many questions using the four rules of calculation on the Move On practice tests CD-ROM, but three from each of Level 1 and Level 2 have been indicated which focus on using subtraction.

It would be helpful for participants to obtain a copy of the CD-ROM (available to order from the Move On web site or from DfES Publications on 0845 60 222 60, quoting reference NTT/PT04/CD) to enable them to practise the various approaches to calculations and to use with their learners.

Level 1

Practice Test	**Question**
Test B	Q 8
Test C	Q 34
Test D	Q 3

Level 2

Note: there are many questions in the Level 2 tests which require two-stage calculations. The questions indicated below focus mainly on subtraction, although may require an understanding of other number-related skills.

Practice Test	**Question**
Test A	Q 12
Test B	Q 15
Test C	Q 16

Activity 6 – Multiplication

Method:

- Quickly review the TERPA acronym and its process, explaining that the same format will be used to look at multiplication.

- **Try**
  - Write a long multiplication task on flip-chart paper, e.g. 67 x 42, and ask participants to complete the calculation by a method of their own choosing.
  - Ask for a volunteer to demonstrate his/her method, then ask if others did it differently or know a different way and encourage further demonstrations. You may need to check that what is being said by volunteers is correct and comment as necessary!

- **Explore**
  - Pick one to two other methods that have not been shown and demonstrate and explain them (the handout is available as support) on the flip-chart or whiteboard. Check that participants are comfortable with and understand the different methods demonstrated.
  - One particular method which research has identified is the gypsy method. However, this has not been included on the handout, but it could be demonstrated at this stage for participants' interest if appropriate.

- **Reinforce**
  - Distribute Handout 7 detailing all the methods and ask participants to add on any other methods demonstrated (if any).
  - Look through the handout together and invite questions/comments on the various methods. You could elicit from participants what approaches they have come across in their work with learners and, if time permits ask them to demonstrate.

- **Practise**
  - Ask participants to pair up. Using Handout 8, each participant should demonstrate his/her preferred method to the other person and then attempt the same calculation using at least two different methods to the original method. Suggest that they attempt methods that are completely new to them where possible. Encourage discussion of the new methods within the pair, including what they find harder/easier/confusing/etc about the new methods attempted.
  - **Note**: Remind participants to use the correct terminology.

- **Assess**
  - Ask participants to apply their learning to practice questions taken from the Move On practice tests CD-ROM.
  - **Note**: This will require participants to practise the skill of converting a problem in words into a multiplication task and link back to the language of maths work in earlier activities.

- **Try it out with your learners!**
  - See 'Try it out with your learners! Activity 6' for a multiplication game.

Handout 7 – Multiplication

Repeated addition

e.g. 36 x 4 = 36 + 36 + 36 + 36
= 144

Note: This method becomes unwieldy for larger numbers, so learners who choose this method should be introduced to alternatives.

Number facts

Times tables, e.g. 6 x 8 = 48

Learners usually have most difficulty in mastering the 6, 7, 8 and 9 times tables. The gypsy method can be of great use in this situation, but needs to be demonstrated in person as diagrams are difficult to follow.

Learners who are not confident with times tables should be encouraged to use a multiplication square, gradually working towards familiarity with all the times tables up to 10 x 10. The square can also be used to look for patterns in the tables.

A learner who is unlikely to ever master times tables can sometimes be taught, with practice, to produce a multiplication square within three minutes, perhaps at the beginning of an exam. The square is then available to refer to throughout the exam.

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Learners should be encouraged to look for shortcuts, e.g.:

- to multiply by 4, double and then double again
- to multiply by 10, move the decimal point one place to the right
- to multiply by 5, multiply by 10 and then halve.

Partitioning and factorising

e.g. (1) 20 x 4 = (10 x 4) + (10 x 4)
= 40 + 40
= 80

e.g. (2) 43 x 6 = (40 x 6) + (3 x 6)
= (10 x 6) + (10 x 6) + (10 x 6) + (10 x 6) + (3 x 6)
= 60 + 60 + 60 + 60 + 18
= 240 + 18
= 258

e.g. (3) 43 x 6 = (40 x 6) + (3 x 6)
= (4 x 6 x 10) + (3 x 6)
= 240 + 18
= 258

For decimals:

e.g. (4) 3.6 x 4 = (3 x 4) + (0.6 x 4)
= 12 + (6 x 4 ÷ 10)
= 12 + 2.4
= 14.4

e.g. (5) 0.4 x 0.03 = (4 ÷ 10) x (3 ÷ 10 ÷ 10)
= 4 x 3 ÷ 10 ÷ 10 ÷ 10
= 12 ÷ 10 ÷ 10 ÷ 10
= 0.012

e.g. (6) 23.42 x 0.63 = 2342 x 63 ÷ 10 ÷ 10 ÷ 10 ÷ 10
= 147546* ÷ 10 ÷ 10 ÷ 10 ÷ 10
= 14.7546

***Note**: One of the following methods will be needed to find 2342 x 63.

For example, using the grid method:

	2000	30	40	2	
60	120000	18000	2400	120	140520
3	6000	900	120	6	and on the
	126000	18900	2520	126	other hand

The following two methods use the principles of partitioning.

Egyptian method

e.g. 63 x 37

63 x 1 = 63
63 x 2 = 126
63 x 4 = 252
63 x 8 = 504
63 x 16 = 1008
63 x 32 = 2016

To make 37 it is necessary to use the emboldened multiples of 63: 32, 4 and 1.

e.g.

2016
252
63 +
2331

10, 5, 2 Method

This is similar to the Egyptian method except that only the multiples 10, 5, 2 and 1 are used as any number can be made up of these (as with currency).

e.g. 63 x 38 = (63 x 10) + (63 x 10) + (63 x 10) + (63 x 5) + (63 x 2) + (63 x 1)
= 630 + 630 + 630 + 315 + 126 + 63
= 2394

Grid method

e.g. 153 x 54

	100	50	3	
50	5000	2500	150	7650
4	400	200	12	612 +
	5400	2700	162	8262

Notes:

1 The subtotals may be added horizontally (as above), vertically

5400
2700
162 +
8262

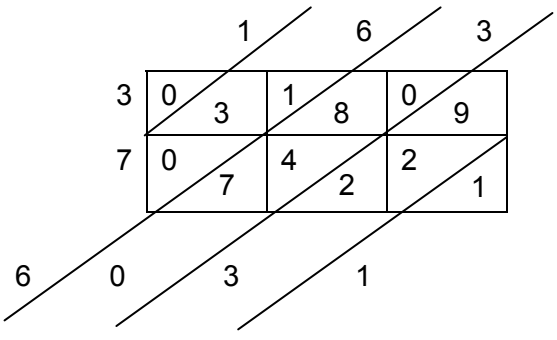
or diagonally:

5000
2900
350
12 +
8262

2 This method is a good introduction to written multiplication methods because it emphasises the importance of the place value of each digit.

Lattice method

e.g. 163 x 37

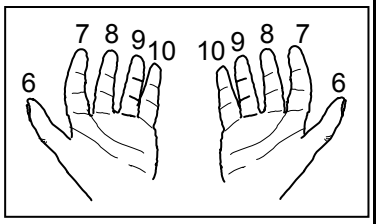


Note: The digits within the lattice are added diagonally, moving from the right to the left and 'carrying' digits to the next column as required.

Handy multiplication

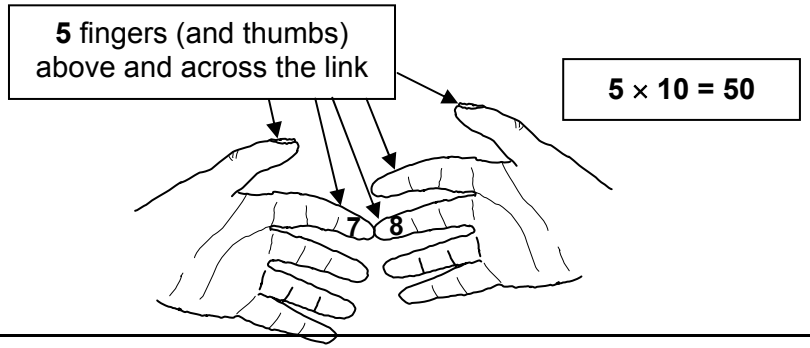
This is a method for finding the harder multiplication facts from the six, seven, eight and nine times tables. It involves memorising a routine – but the routine is something to do, rather than something to say, so it may be helpful for kinaesthetic learners who can remember movements more easily than words.

The thumb and fingers of each hand are first labelled with the numbers 6 to 10.



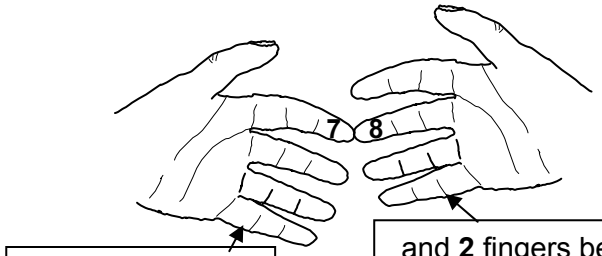
Then the tips of the two fingers whose numbers are to be multiplied are brought together so they are just touching.
For example, to multiply 7 by 8 the tip of the forefinger (labelled 7) must just touch the tip of the middle finger (labelled 8) to form a link.

Now the two touching fingers, and all the fingers (and the thumbs) above them, are counted, giving 2 on one hand and 3 on the other – or 5 altogether. This is the number of *tens* in the total product.



Next we look at the fingers below the link. There are 3 on one hand and 2 on the other. These two numbers are multiplied together, and the product, 6, is added to the 5 tens that we already have.

So 7 times 8 is *5 tens*, *plus 6* – or 56 altogether.



If you want to see how *Handy Multiplication* works, then the formula you need is:

$$a \times b = ((a - 5) + (b - 5)) \times 10 \quad + \quad (10 - a) \times (10 - b)$$

The number of fingers (and thumbs) touching and above on one hand…

and on the other hand

The fingers below on one hand…

and on the other hand

It is unlikely that many learners will be able to follow this algebra – but for some, the movements can be memorised and recalled much more easily than the recited chants of the multiplication tables.

Adapted from: Tandi Clausen-May, *Teaching maths to pupils with different learning styles*, London Sage, 2005 (in press)

Short multiplication

e.g. (1) 45 x 4

```
  45
   4  x
  20  (5 x 4)
 160  (40 x 4)
 180
```

Note: If a learner is moving to this method from the grid method it might be more logical to multiply 40 x 4 on the first line and then 5 x 4 on the second line.

Long multiplication

e.g. (2) 273 x 24

```
  273
   24  x
 1092  (273 x 4)
 5460  (273 x 20)
 6552
```

Notes:

1 It is important to emphasise to learners the need to work from right to left, multiplying each digit of the first number by 4 ('carrying' digits into the next column as required), then placing a 0 in the furthest right position on the second line and then multiplying each digit of the first number by 2 ('carrying' digits into the next column as required). Finally, the two lines of multiplication should be added together, moving from right to left and 'carrying' into the next column as required.

2 When this method is carried out in the traditional way, the use of the 0 on the second line requires some explanation. The first line of multiplication is x 4, but the second line, although it might appear to be x 2, is actually x 20.

3 Take care over the positioning of the 'carrying' digits as learners will have different preferences.

Multiplication problems in words

It is important that learners are taught how to convert a problem presented in words into a multiplication task which can be solved by one of the methods above.

e.g. The monthly repayments on a new television are £29.32 for 24 months.

Calculate the total cost of the television.

29.32 x 24 = 2942 x 24 ÷ 10 ÷ 10 = £703.68

It is always advisable to encourage learners to estimate the answer in order to check that the answer obtained is reasonable. The use of inverse operations could also be used to check the answer. This helps in establishing the link between multiplication and division.

The methods demonstrated above include the ones most commonly encountered and those suggested in the Adult Numeracy Core Curriculum, but it is possible that you will encounter additional methods or variations on these methods.

Handout 8 – Multiplication examples

With a partner work out the following question using three different methods. Talk it through as your partner listens and then reverse roles to tackle it another way.

463 x 164

Method 1

Method 2

Method 3

Trainer notes 3 – Practice questions

Appropriate questions for multiplication from the practice tests are referenced below. There are many questions using the four rules of calculation on the Move On practice tests CD-ROM, but three from each of Level 1 and Level 2 have been indicated which focus on using multiplication.

It would be helpful for participants to obtain a copy of the CD-ROM (available from the Move On web site or to order from DfES Publications on 0845 60 222 60, quoting reference NTT/PT04/CD) to enable them to practise the various approaches to calculations and to use with their learners.

Level 1

Practice test	**Question**
Test A	Q 3
Test B	Q 16
Test C	Q 6

Level 2

Note: there are many questions in the Level 2 tests that require two-stage calculations. The questions indicated below focus mainly on multiplication, although may require an understanding of other number-related skills.

Practice test	**Question**
Test A	Q 29
Test B	Q 26
Test C	Q 21

Activity 7 – Division

Method:

- Quickly review the TERPA acronym and its process, explaining that the same format will be used to look at multiplication.

- **Try**
  - Write a long division task on flip-chart paper, e.g. 2282 ÷ 14 and ask participants to complete the calculation by a method of their own choosing.
  - Ask for a volunteer to demonstrate their method, then ask if others did it differently or know a different way, and encourage further demonstrations. You may need to check that what is being said by volunteers is correct ,and comment as necessary!

- **Explore**
  - Pick one to two other methods which have not already been shown and demonstrate and explain them (the handout is available as support) on the flip-chart or whiteboard. Check that participants are comfortable with and understand the different methods demonstrated.

- **Reinforce**
  - Distribute Handout 9, detailing all the methods, and ask them to add on other methods which have been demonstrated (if any).
  - Look through the handout together and invite questions/comments on the various methods. You could elicit from participants what approaches they have come across in their work with learners and, if time permits ask them to demonstrate.

- **Practise**
  - Ask participants to pair up. Using Handout 10 each pair should demonstrate the same calculation using at least two different methods for each calculation. Suggest that they attempt methods which are completely new to them where possible. Encourage discussion of the new methods within the pair, including what they find harder/easier/confusing/etc about the new methods attempted.
  - **Note**: Remind participants to use the correct terminology.

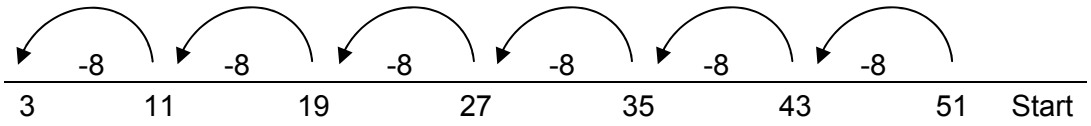
- **Assess**
  - Ask participants to apply their learning to practice questions taken from the Move On practice tests CD-ROM.
  - **Note**: This will require participants to practise the skill of converting a problem in words into a division task and link back to the language of maths work in earlier activities.

Handout 9 - Division

Division can be thought of as sharing or as repeated subtraction, i.e. 51 ÷ 8 means 'how many times can 8 be subtracted from 51?'

It might be necessary when introducing the concept of division to use concrete examples, e.g. sharing 36 counters between six students by issuing them one at a time to each student until there are none left.

Division can be demonstrated using a number line, e.g.



so, 51 ÷ 8 = 6 r3 or 6⅜

Note: It is important that learners are taught how to deal with remainders. Remainders can be expressed simply as a remainder or as a fraction or decimal. How to deal with remainders in real life examples of division should also be explained. For example, 'How many coaches each seating 50 are required to take 230 people on a trip?' requires the student to work out 230 ÷ 50 = 4 r30, but the remainder of 30 means that a 5th coach will be required.

Number facts

Familiarity with times tables enables learners to carry out mental division:

e.g. if 5 x 7 = 35, then 35 ÷ 7 = 5 and 35 ÷ 5 = 7

Learners should be encouraged to look for shortcuts, e.g.:

- to divide by 4, halve and halve again
- to divide by 10, move the decimal point one place to the left
- to divide by 5, divide by 10 and then double.

Repeated subtraction for larger numbers

This method combines the method of repeated subtraction with knowledge of number facts, which enables the subtraction of multiples of the dividing number. This is sometimes called 'chunking'. Examples:

133 ÷ 4

133
40 - (10 x 4)
93
40 - (10 x 4)
53
40 - (10 x 4)
13
12 - (3 x 4)
1

so, 133 ÷ 4 = 33 r1 or 33¼

This method can start off by using the conventional division notation, e.g.

442 ÷ 17

$17\overline{)442}$

170 - (10 x 17)
272
170 - (10 x 17)
102
85 - (5 x 17)
17
17 - (1 x 17)
0

so 442 ÷ 17 = 26 i.e. 17 can be subtracted from 442 twenty-six times

Note: Clearly it is easier, and makes the process quicker, to begin by subtracting multiples of 10.

Partitioning and factorising

e.g. (1) 600 ÷ 20 = 600 ÷ 10 ÷ 2
= 60 ÷ 2
= 30

e.g. (2) 36000 ÷ 40 = 36000 ÷ 10 ÷ 10 ÷ 10 ÷ 4
= 36 ÷ 4
= 9

For decimals (multiplying both sides by multiples of 10):

e.g. (3) $54 \div 0.9 = 540 \div 9$
$= 54 \div 9 \times 10$
$= 6 \times 10$
$= 60$

e.g. (4) $23.6 \div 0.02 = 2360 \div 2$
$= 1180$

Short division

e.g. $6\overline{)324}$ 54

Commentary:
- how many times does 6 divide into 3? The answer is 0 r3, so the remainder 3 is carried in front of the 2.
- how many times does 6 divide into 32? The answer is 5 r2, so the 5 is written above and the remainder 2 is carried in front of the 4.
- how many times does 6 divide into 24? The answer is 4 r0, so the 4 is written above and there is no remainder to be carried.

For decimals:

e.g. (1) $6\overline{)32.4}$ 5.4

e.g. (2) $4\overline{)185.00}$ 46.25

Notes:

1 In this second example, the 0's have been added to enable the division to continue to give the remainder as a decimal.

2 It is important when using this method of division that the numbers of the number to be divided and the answer are correctly lined up according to place value. This is particularly important when dividing decimals.

3 The use of a 0 as a place holder should be specifically taught:

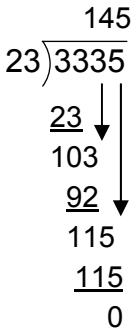
e.g. $4\overline{)824}$ 206

Long division by the short division method

This works exactly as above, except that the remainder could have two or more digits, which require to be carried. It is therefore necessary to set out the calculation allowing plenty of space.

Long division

e.g.



Commentary:

- 23 divides into 33 once and 1 x 23 = 23, so 1 is written on the answer line and 23 is written below 33 and subtracted from it.
- In the answer 10 is added the next digit from the number to be divided which is 3, making 103.
- 23 divides into 103 four times and 4 x 23 = 92, so 4 is written on the answer line and 92 is written below 103 and subtracted from it.
- In the answer 11 is added the next digit 5, making 115.
- 23 divides into 115 five times and 5 x 23 = 115, so 5 is written on the answer line and 115 is written below 115 and subtracted from it, leaving 0.

Notes:

1 Again, it is important to keep all digits lined up correctly according to place value.

2 Learners tend to find this the most difficult method of division, but older learners might be more familiar with it.

Division problems in words

It is important that learners are taught how to convert a problem presented in words into a division task which can be solved by one of the methods above.

e.g. A room of height 2.5 metres is to be wallpapered. How many lengths of wallpaper can be obtained from a roll of wallpaper 14 metres long and how much wallpaper will be left over? Assume that there is no pattern match to be taken into account.

$$14 \div 2.5 = 140 \div 25 = 5\frac{15}{25} = 5\frac{3}{5}$$

i.e. five lengths of wallpaper can be obtained from the roll with three fifths of a length (or 1.5 metres) left over.

It is always advisable to encourage learners to estimate the answer in order to check that the answer obtained is reasonable. The use of inverse operations could also be used to check the answer. This helps in establishing the link between multiplication and division.

The methods demonstrated above include the ones most commonly encountered and those suggested in the Adult Numeracy Core Curriculum, but it is possible that you will encounter additional methods or variations on these methods.

Handout 10 – Division examples

With a partner, work out the following questions, using two different methods for each. Talk it through as your partner listens, and then reverse roles to tackle it another way.

1 2432 ÷ 19

2 247.5 ÷ 15

Method 1

1

2

Method 2

1

2

Trainer notes 4 – Practice questions

Appropriate questions for division from the practice tests are referenced below. There are many questions using the four rules of calculation on the Move On practice tests CD-ROM, but three from each of Level 1 and Level 2 have been indicated, which focus on using division.

It would be helpful for participants to obtain a copy of the CD-ROM (available to order from the Move On web site or from DfES Publications on 0845 60 222 60, quoting reference NTT/PT04/CD) to enable them to practise the various approaches to calculations and to use with their learners.

Level 1

Practice Test	Question
Test A	Q 8
Test B	Q 7
Test D	Q 4

Level 2

Note: there are many questions in the Level 2 tests that require two-stage calculations. The questions indicated below focus mainly on division, although may require an understanding of other number-related skills.

Practice Test	Question
Test B	Q 25
Test C	Q 15
Test D	Q 10

Activity 8 – FAQs (frequently asked questions)

- Using Handout 11, ask participants in small groups to discuss and agree some answers to the example FAQs on the handout. Suggest they add FAQs from their own learners and discuss and agree the appropriate responses to them.

- Facilitate a whole-group feedback on the responses to FAQs on Handout 11, and list any new FAQs that have been identified by the whole group, agreeing the responses. Refer participants to Handout 12 (answers), which has suggested responses for the FAQs on Handout 11.

- Explain that any new FAQs and agreed responses will be sent to Move On to include on the web site.

Handout 11 – FAQs

FAQ 1 Why do you put the '0' on the second line of a long multiplication?

FAQ 2 What does 'guzzinter' (goes into) mean?

FAQ 3 What's the point of being able to do long multiplication or division when you can just use a calculator?

FAQ 4 Why must you work from right to left for +, - and x sums?

Handout 12 – FAQ answers

FAQ 1 Why do you put the '0' on the second line of a long multiplication?

A. e.g. 273 x 24

```
 273
  24  x
1092  (273 x 4)
5460  (273 x 20)
6552
```

The first line of multiplication is x 4, but the second line, although it might appear to be x 2, is actually x 20 since the 2 is in the tens column. The 0 moves each digit one place to the left, i.e. increasing its value tenfold.

FAQ 2 What does 'guzzinter' (goes into) mean?

A. The concept of division can be difficult to explain to a learner and the traditional question asked whilst attempting a division calculation, 'How many times does 4 go into 36?' can be particularly difficult to understand. The learner needs to be taught that division is the inverse operation to multiplication so that a better question would be, 'What must 4 be multiplied by to give an answer of 36?'

Division can often be better explained as a method of sharing and this approach lends itself to explanation by the use of concrete examples, e.g. If 36 sweets are shared between 4 children how many sweets will each child receive?

A further approach is the idea of repeated subtraction, which is explained on the 'Division' handout.

FAQ 3 What's the point of being able to do long multiplication or division when you can just use a calculator?

A. You won't always have a calculator to hand.

Some exams will not allow a calculator to be used.

Even when a calculator is used it is useful to be able to calculate even just a rough estimate of the answer so that the answer given by the calculator can be checked to determine whether it is sensible (in case the wrong button has been pressed, the batteries are low, etc.).

FAQ 4 Why must you work from right to left for +, - and x sums?

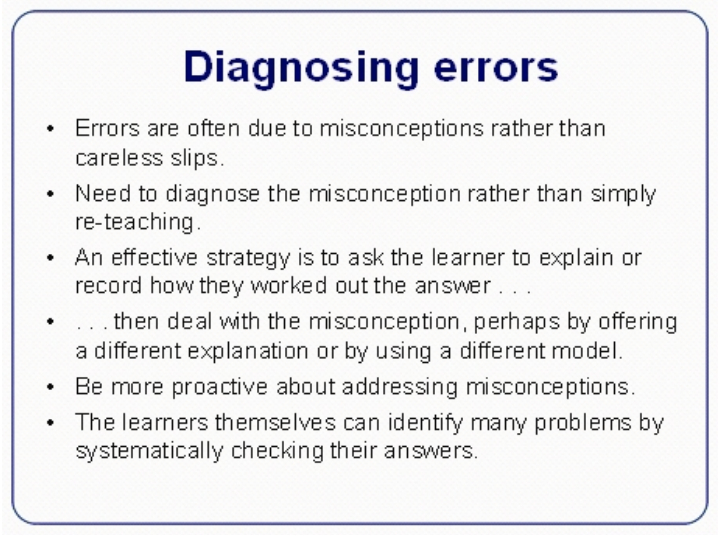
A. When numbers are arranged for vertical calculations, the units column is on the right, the next column moving left is tens, the next is hundreds, etc., so the place value of the digits is ten times greater for every move to the left. If addition and multiplication calculations begin with the units, any multiples of ten can be carried into the next column, and so on for the other columns. With subtraction calculations by decomposition, ten units can be added to the digit in the units column by reducing the digit in the tens column by one, and so on for the other columns.

For a learner to have a clear awareness of what is actually happening, it is important to teach place value before the four operations.

Activity 9 – Common errors and misconceptions[2]

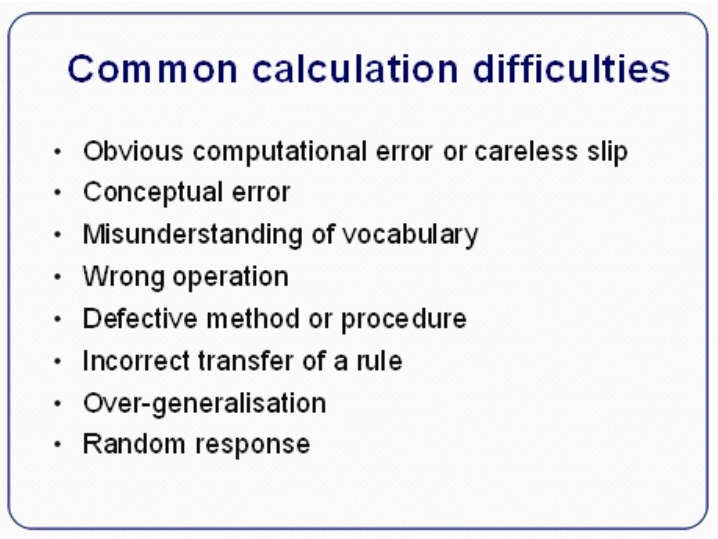
Method:

- Use Slide 5 to introduce the activity. Explain each bullet point and stress that if learners have a preferred method (or indeed, any method!) for completing an operation, then try to determine where they are having a problem and support them to overcome it. **Don't try to teach a learner a different method to complete a calculation unless it seems appropriate**, e.g. use of number lines for addition if the learner uses columns or horizontal methods.



- Use Slide 6 to outline the common calculation difficulties learners may have and refer participants to Handout 13.

- Ask participants, in small groups, to complete an error analysis on one of the learner answers in Handout 14.



- Ask for brief feedback and refer participants to Handout 15 for reference.

- Mention that error analysis information and examples have been taken from the primary numeracy strategy/Key Skills Support Programme/*Skills for Life* diagnostic assessment resources. There are further examples of learner errors in the *Skills for Life* numeracy diagnostic assessment.

[2] Taken from the *Primary Numeracy Strategy* and *Key Skills Support Programme* materials

Handout 13 - Common calculation difficulties

1 Obvious computational error or careless slip

Correct operation but incorrect recall of basic number facts.

Example: A pencil cost 37p and a ballpoint pen costs 45p. How much would eight pencils and three ballpoint pens cost altogether?

Answer:

$$\begin{array}{r} 8 \times 37 = 3\ 0\ 3 \\ 3 \times 45 = \underline{1\ 3\ 5}\ + \\ 4\ 3\ 8 \end{array}$$

2 Conceptual error

Student has not grasped the concept of the relevant operation.

Example: Calculate the value of 5^2

Answer: 10

3 Lack of understanding of vocabulary

Misinterpretation of vocabulary (especially when English is not first language).

Example: Estimate the value of 28.93 x 20.987

Answer: 607.15391

4 Wrong operation

The wrong operation is used in the solution.

Example: Find 15% of £300

Answer: $\frac{300}{15} \times 100 = 2000$

5 Defective procedure or method

Correct operation chosen, no number fact errors, but errors in carrying out steps of the procedure.

Example: There are 12 boys and 18 girls in a Key Skills class. What percentage of the whole class are boys?

Answer: $\frac{12}{30}$ x 100 = 250%

6 Over-generalisation

Pattern or rules learned and then applied to situations where they no longer[3] work.

Example: The rule learned is 'to multiply by 10, add a nought'.
What is 3.1 x 10?

Answer: 3.10

7 Under-generalisation

Too few examples and so generalises on the basis of too little data.

Example: Yesterday, the temperature was 5°C lower than the day before. The temperature yesterday was 15°C. What will the temperature be tomorrow?

Answer: 10°C

8 Random response

A wild guess!

Example: A coach has been hired for a college trip. It costs £600 for the day. If there are 50 students on the trip, how much should each student pay?

Answer: £30

[3] Examples taken from the Key Skills Support Programme materials

Handout 14 - Diagnosing errors in number[4]

Here are some example calculations that a learner has completed.

Look at the answers and decide what errors have been made and why.

What remedial action do you think is needed in each case?

Example 1

20
13 -
17

Example 2

$^{7}8^{1}\cancel{2}$
− 4 0
3 2

Example 3

Problem: Angela thinks of a number. She doubles it and then adds 5. The answer is 19. What is her number?
Learner's answer: 14

Example 4

3525 divided by 5 = 75

Example 5

0.6 – 0.25 = 0.45

[4] Examples taken from the DfES *Numeracy Diagnostic Assessment* materials

Handout 15 - Diagnosing errors in number[5]

Example 1

20
13 -
17

Commentary: This learner has not recognised that it is necessary to 'borrow' and 'pay back' – if this is how they have been taught (common with older learners).

Example 2

$^{7}\not{8}\,^{1}2$
4 0 -
3 2

Commentary: This learner has been doing a whole set of sums where the decomposition method has been used to work them out. He has then continued to apply the method even when the sum does not require it. This is an example where the learner can be encouraged to look at the problem before they start and to think of alternative ways to solve it, rather than relying on a 'method' which may be inappropriate for the size and type of numbers involved.

Example 3

Problem:
Angela thinks of a number. She doubles it and then adds 5. The answer is 19. What is her number?

Learner's answer: 14

Commentary: The 'language' has definitely got in the way for this learner. S/he has seen the two numbers in the question and subtracted them. The concept of working backwards to get to the answer is partly in place, but 'overload' was probably quickly reached when the concept of doubling was introduced in this 'wordy' problem.

Example 4

3525 divided by 5 = 75

Commentary: This learner has not used the zero as a place holder. This is probably a difficulty with place value. The learner has performed a mechanical operation without an understanding of the operation or a 'feel' for the size of the numbers. With the latter a learner should be able to recognise that you do not obtain a two-digit answer when dividing a four-digit number by one single digit.

Example 5

0.6 – 0.25 = 0.45

Commentary: The learner has failed to use zero as a place holder.

[5] Examples taken from the DfES *Numeracy Diagnostic Assessment* materials

Handout 16 - Action Plan

Action	Outcome	Who would be responsible?	Comment

Handout 17 'Try it out with your learners' activities

Try it out with your learners! Activity 1

Use the meaning cards with your learners in a matching activity. Focus on number words and the associated meanings that learners will need in their core course work.

Use the activity to introduce learners to a new topic at the start of a session. You could also use this type of activity to introduce learners to new words and definitions in any topic or area of study.

Try it out with your learners! Activity 2

The questions in the adult numeracy tests do not ask learners to complete straightforward 'sums', but require them to be able to complete number problems couched in a range of terms such as those identified in the previous activities.

Use a similar activity on the language of maths with your learners to generate a discussion on the language they use in their number calculation methods.

This will provide them with an opportunity to think about the different ways problems can be presented to them in their work and in the adult numeracy tests.

Try it out with your learners! Activity 3

Use the 'Follow Me' cards with learners, focussing on a particular area or topic relevant to the teaching and learning planned.

Time the sessions to encourage learners to speed up their mental maths, as this will help them in their course work and in the adult numeracy tests.

To encourage learner involvement, you could set them a task to develop a set of 'Follow Me' cards to use in whole group sessions.

Try it out with your learners! Activity 4

Give out the digit cards 1-9 and ask the learners if they can make a magic 3 x 3 square where each column, row and diagonal totals 15.

This can be extended to a 4 x 4 magic square where the rows, columns and diagonals total 34.

Try it out with your learners! Activity 5

Subtraction bingo

This can be adapted depending on the ability of the learners. Instead of calling out a number, the caller reads out a sum or a word question, which needs a subtraction calculation to get the correct answer. If the learners have that answer on their card they cover it up. The winner is the first to cover a line or the whole card.

Try it out with your learners! Activity 6

Fizz Buzz

This activity is a way of practising multiplication facts. Ask your group of learners to count to 100 (or more), going around the group with each person (including the tutor) saying one number, the first person saying ‘1’, the second ‘2’ etc. If the number is a multiple of 3 the learner says ‘Fizz’ instead of the number, if the number is a multiple of 5 the learner says ‘Buzz’ instead of the number and if the number is a multiple of 3 and 5 the learner says ‘Fizz Buzz’. Multiples of other numbers can be used to practise other multiplication tables.